Finite element modelling of a 2D induction heating problem

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Challenges of modelling induction heating

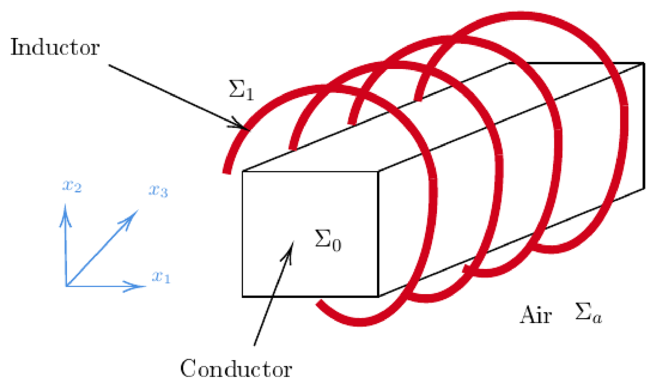

Fig. 1: 3D induction heating domain.

Induction heating is a method of heating metal that is widely used in industry. One difficulty in building a finite element model is the **steep gradients** in magnetic field and temperature due to the skin-effect, which depends on the frequency of the current.

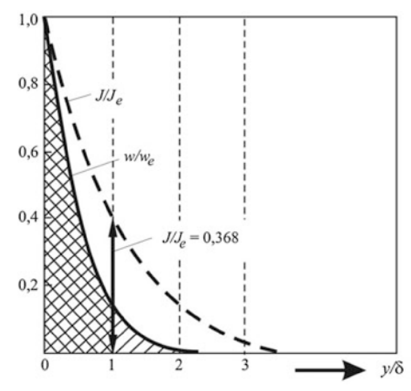

Fig. 2: C42 mod-microalloyed steel has a skin depth of about 40μm for a frequency of 100kHz. Image from [2].

Induction heating equations

Let Ω be the 2D cross section of the billet with boundary $\partial\Omega$. Denote the temperature by u, and magnetic field strength by H. Solve the **magnetic equation**

$$-\text{div}(\sigma^{-1}(u)\nabla H) + i\omega\mu(u)H = 0 \qquad \text{in } \Omega,$$
$$H = H_0 \qquad \text{on } \partial\Omega,$$

and the **heat equation**

$$\rho(u)C_p(u)\frac{\partial u}{\partial t} - \text{div}(\kappa(u)\nabla(u)) = \frac{1}{2\sigma(u)}|\nabla H|^2 \qquad \text{in } \Omega,$$
$$-\kappa(u)\frac{\partial u}{\partial n} = \alpha(u|u|^3 - u_{\text{amb}}^4) + \beta(u - u_{\text{amb}}) \qquad \text{on } \partial\Omega.$$

Here, σ, μ, ρ, C_p and κ are material properties depending on u, H_0 is the magnetic field strength in air, u_{amb} is the ambient temperature, ω is the current frequency, and α and β are the radiative and convective coefficients, respectively [1].

Effect of the Curie point

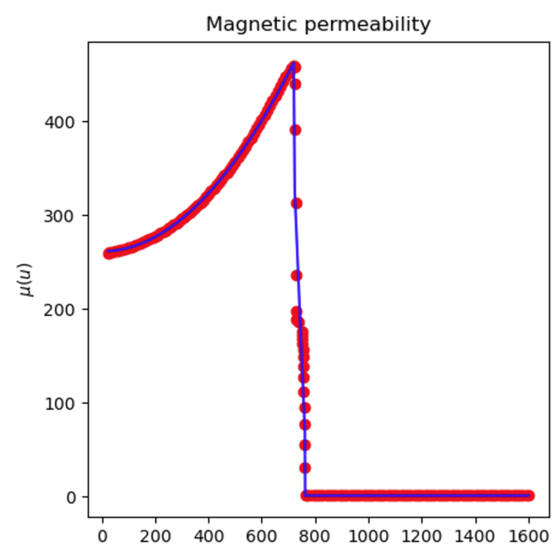

Fig. 5: Relative permeability of C42 mod-microalloyed steel from 25 − 1600°C.

When the temperature reaches the **Curie point** there is a steep discontinuity in the magnetic permeability. This adds stiffness to the systems of equations and makes the convergence of the FEM challenging.

Experiment setup

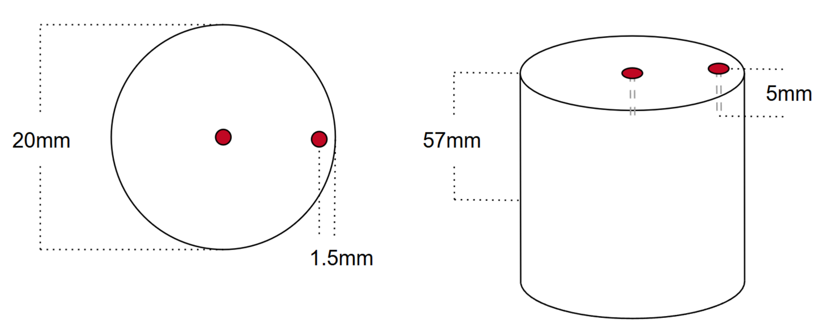

Fig. 3: The piece of C42 mod-microallyed steel used in the experiment.

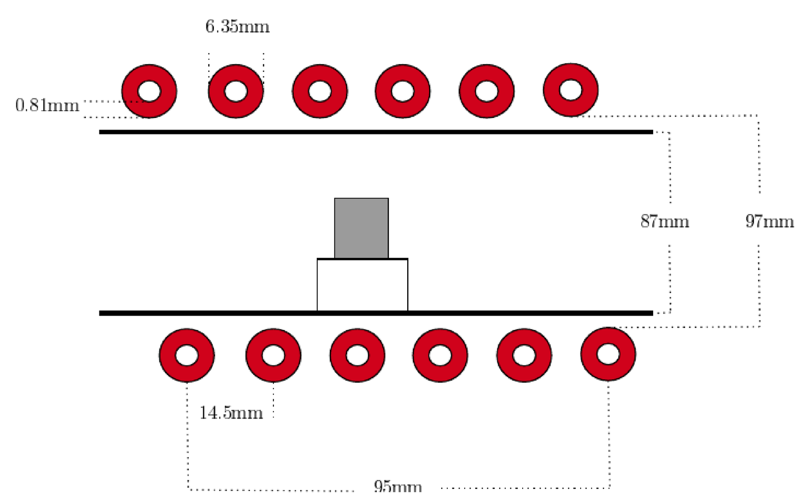

Fig. 4: Cross-sectional layout of the inductor and instrumented sample.

Here we run an experiment with C42 mod-microalloyed steel sample and compare the model to experimental data. The frequency is 95kHz with skin depth of 42μm.

Choice of mesh

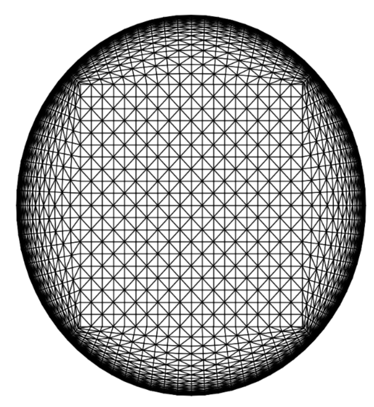

Fig. 6: An anisotropic mesh.

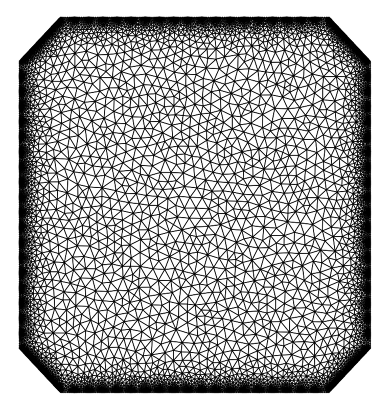

Fig. 7: A conforming mesh.

Anisotropic meshes capture the skin-effect, but the **irregular shape** of the elements can cause problems. **Conforming meshes** can handle more complex geometries but are more **computationally expensive**.

Experiment results

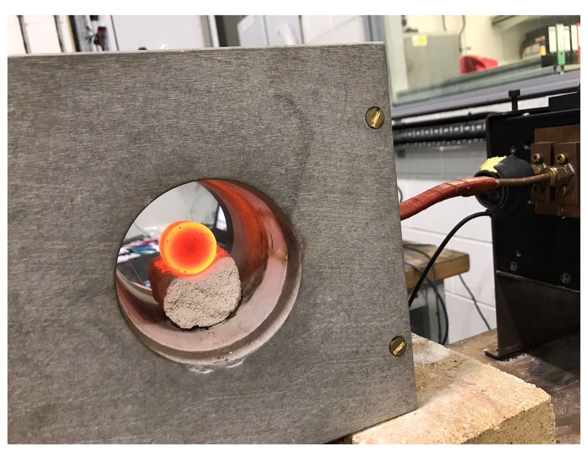

Fig. 8: Photograph of the experiment.

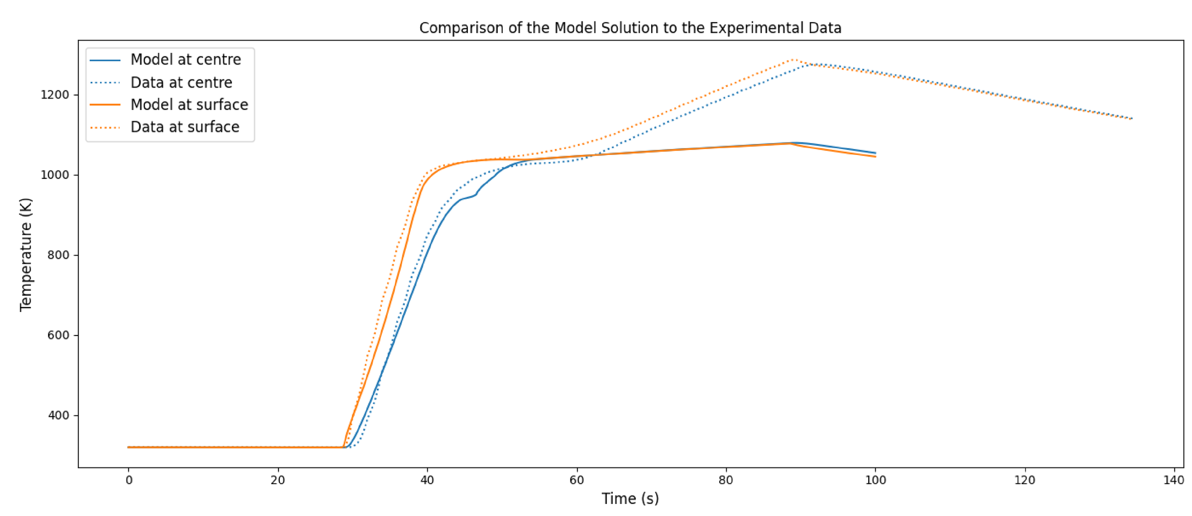

Fig. 9: Results from the experiment.

The results show that this model has a good fit to the experimental data until it reaches the Curie point. After that, at the Curie point, the discontinuity in the physical coefficients creates a locking phenomenon that does not allow the temperature to increase.

References

[1] R. Touzani and J. Rappaz. *Mathematical Models for Eddy Currents and Magnetostatics: with Selected Applications.* (2014) Springer Nature.

[2] S. Lupi, M. Forzan, and A. Aliferov. *Induction and Direct Resistance Heating: Theory and Numerical Modelling.* (2015) Cham: Springer International Publishing AG.

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